

Re-Examination Statistical Methods in Physics

Wednesday, June 27 2018

Before you start, read the following:

- Write your name and student number on top of each page of your exam;
- Illegible writing will be graded as incorrect;
- Annexes:
 - Integral of the Standard Normal distribution
 - Quantiles of the t distribution
 - Binomial distribution
 - Quantiles of the Chi-squared distribution

Problem 1 (20 Points)

Let X_1, \dots, X_n be a random sampler from a distribution with pdf of $f(x) = (2\beta - 2x)/\beta^2$ for $x \in (0, \beta)$ and $f(x) = 0$ otherwise. Please find estimator for β , or equation which the estimator satisfy using two different methods.

Problem 2 (20 Points)

Let X_1, \dots, X_n be a random sampler from a distribution with pdf of

$$f(x) = \frac{1}{\pi(1 + (x - \beta)^2)}$$

for $x \in (-\infty, \infty)$.

- a) Please find estimator for β , or equation which the estimator satisfy using MLE method. **(15 points)**
- b) Is it possible to estimate β using MME? Provide solid arguments. **(5 points)**

Problem 3 (25 Points)

Let X_1, \dots, X_n be a random sampler from a distribution with pdf of $f(x) = (2x/\eta^2) \exp(-x^2/\eta^2)$ for $x > 0$ and $f(x) = 0$ otherwise. Find the uniform most powerful test (test statistic and describe how to get the critical region) for hypothesis $H_0: \eta = \eta_0$ versus $H_1: \eta < \eta_0$.

Problem 4 (15 Points)

A random sample of 20 observations gave the following summary statistics: $\sum x_i = 234$ and $\sum x_i^2 = 3048$. Assuming that the data may be looked upon as a random sample from a normal population, construct a 95% confidence interval for the actual average, μ .

Problem 5 (20 Points)

Suppose we have a random sample of size 25 from normal population with an unknown mean μ and a standard deviation of 4. We wish to test the hypothesis $H_0: \mu = 10$ versus $H_1: \mu > 10$. Let the rejection region be defined by: reject H_0 if the sample mean $\bar{x}_{25} > 11.2$.

- Find α .
- Find β for $H_1: \mu = 11$.
- What should the sample size and critical region be if $\alpha = 0.01$ and $\beta = 0.2$ assuming $H_1: \mu = 11$?